Acceleration of a Spherical Particle in a Uniform Gas Flow

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Nomenclature

A = nozzle area
C_D = particle drag coefficient
H_o = gas stagnation enthalpy
m, $\dot{m}$ = mass and mass flow rate
M = Mach number
p_o = total pressure
R = radius
S = reference area, πR_p^2
t = time
V = velocity
x = distance from nozzle throat
ρ = density

Subscript

$(\ )_f$ = nozzle exit
$(\ )_g$ = gas
$(\ )_i$ = initial
$(\ )_p$ = particle

Superscript

$(\ )^*$ = sonic flow conditions

Introduction

IN developing design criteria for facilities to simulate the effect of particle erosion on bodies penetrating dust clouds a great many tradeoffs have to be made between nozzle geometry, working fluid, and particle size. Although computer programs[1] are available for computing particle acceleration in an expanding flow, such a procedure is very time consuming when doing parametric studies. Approximate analytical solutions are given in the present Note which allow parametric studies to be performed with a minimum of effort.

Analysis and Results

The basic drag equation for a particle in a gas flow is

$$m_p dV_p/dt = \tfrac{1}{2}\rho_g(V_g - V_p)^2 C_D S_p \tag{1}$$

Assuming the particle to be spherical and letting $V_p dt = dx$, $\rho_g V_g = \dot{m}_g/A$; $K_1 = (3C_D/8)(x_f/R_p)(\rho_g{}^*/\rho_p)V_g{}^*$ in Eq. (1) yields

$$V_p dV_p/(1 - V_p/V_g)^2 = K_1 V_g d(x/x_f)/A/A^* \tag{2}$$

Equation (2) can be integrated formally but a knowledge of V_g as an analytical function of the nozzle area ratio and length is required to evaluate the right hand side. However, an approximate solution can be found by assuming V_g constant at some appropriate value. As it will be shown in the results, a good approximation is to take V_g equal to the value at the end on the integration step. Integration of Eq. (2) yields

$$(1 - V_p/V_g)^{-1} - (1 - V_{p_i}/V_g)^{-1} + \ln\left(\frac{V_g - V_p}{V_g - V_{p_i}}\right) = \frac{K_1 K_2}{V_g} \tag{3}$$

where $K_2 = \int_0^z dz'/A/A^*$ and $z = x/x_f$

Received October 6, 1971: revision received December 10, 1971.
Index categories: Nozzle and Channel Flow; Multiphase Flows.

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Equation (3) is transcendental and cannot be solved explicitly for V_p; however, by assuming analytical expressions for the area variation in K_2, the effect of nozzle geometry on particle acceleration can be studied in parametric form. Since long residence times are required to attain high particle velocity, nozzles designed as particle accelerators tend to have a large L/D. As a result, simple analytical expressions can usually be found for the area variation along the nozzle. The integral K_2 has been evaluated for three general nozzle shapes which represent most classes of aerodynamic nozzles considered. Linear radius variation (Conical): $R = ax + b$; $K_2 = R^*/R$, square root radius variation (Bell): $R = ax + b$; $K_2 = 2\ln(R/R^*)/(R^2/R^{*2} - 1)$ and quadratic radius variation (Trumpet): $R = (ax + b)^2$; $K_2 = [(R/R^*)^{-3/2} - 1]/3(1 - R/R^*)$.

To show the effect of nozzle geometry on the final particle velocity, write Eq. (3) as

$$K_1/V_{g_{\max}} = (1/K_2)\, V_{gf}/V_{g_{\max}}\, [\beta/(1-\beta) + \ln(1-\beta)] \tag{4}$$

where $V_{pi} \ll V_{pf}$ has been assumed and $\beta = (V_{pf}/V_{g_{\max}})/(V_{gf}/V_{g_{\max}})$.

$$V_{gf}/V_{g_{\max}} = [(\gamma - 1)/2]^{1/2} M_{gf}\{1 + [(\gamma - 1)/2]\, M_{gf}^2\}^{-1/2}$$

Equation (4) becomes

$$K_1/V_{g_{\max}} = (1/K_2)[(\gamma - 1)/2]^{1/2} M_{gf}\{1 + [(\gamma - 1)/2]\, M_{gf}^2\}^{-1/2} \times [\beta/(1-\beta) + \ln(1-\beta)] \tag{5}$$

For high Mach number flows, $V_{g_{\max}} \cong (2H_o)^{1/2}$ is a good approximation. The parameter $K_1/V_{g_{\max}}$ can be calculated as a function of the three variables A/A^*, γ, β and includes the effects of reservoir pressure and enthalpy, kind of working fluid, particle size and material, and nozzle length on final particle velocity.

Equation (5) is shown in parametric form in Fig. 1 for air at a $\gamma = 1.22$. Only two of the assumed nozzle geometries are shown since the particle acceleration of the bell-shaped nozzle fell considerably below the other two. However, all three geometries exhibit a maximum particle velocity for a given $K_1/V_{g_{\max}}$ at a particular nozzle area ratio. In addition, the trumpet-shape class of nozzles are shown to give a higher maximum final particle velocity than the other two shapes considered. This would be expected since such a nozzle shape

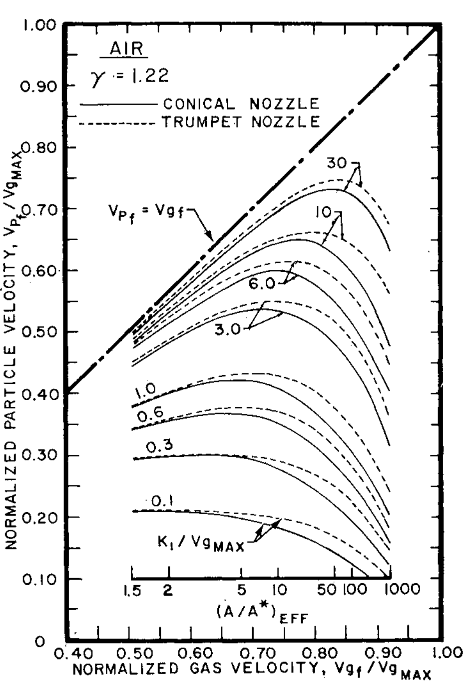

Fig. 1 Effect of nozzle geometry on particle accelleration.

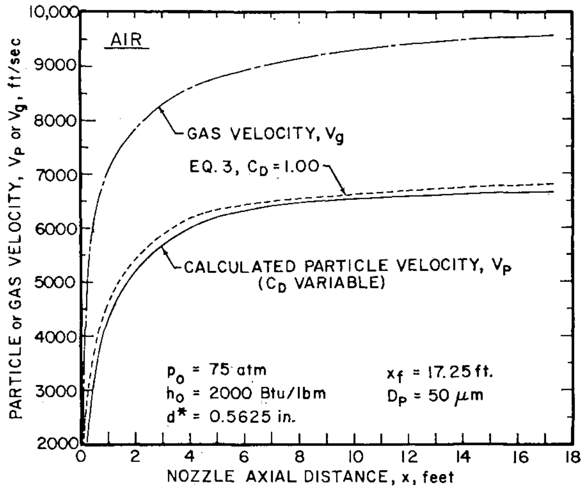

Fig. 2 Comparison of approximate solution in Eq. (3) to a numerical solution for particle velocity along a conical shaped nozzle.

tends to keep the gas density high for a longer time in the nozzle.

To test the accuracy of the approximate solution given in Eq. (3), a comparison with a numerical solution is shown in Fig. 2 where the real nozzle was approximated by a conical nozzle and the gas velocity at the end of each integration step was used in the solution. The agreement is seen to be good except in the nozzle throat region where the rapidly changing gas velocity deviates from the constant gas velocity asumption. In addition, the curves in Fig. 1 were used to evaluate the particle velocity along the nozzle and V_{p_i} was neglected in those curves. Using the same flow conditions and nozzle geometry given in Fig. 2, the effect of particle size on final particle velocity was computed and the results are shown in Fig. 3. Again the agreement between the approximate

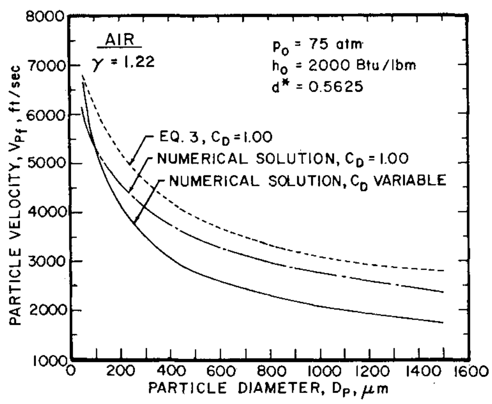

Fig. 3 Effect of particle size on final particle velocity for conditions in Fig. 2.

solution and the exact numerical solution is good with some deviation at the large particle size. All the approximate solutions were carried out with $C_D = 1.0$ which accounts for some of the errors at the large particle sizes. Since, for the larger Reynolds numbers associated with the larger particles C_D decreases below one, computer solutions were carried out for $C_D = 1.0$ and compared with the approximate solution. The agreement is seen to be much better.

Reference

[1] Lorenz, G. C., "Simulation of the Erosive Effects of Multiple Particle Impacts in Hypersonic Flow," *Journal of Spacecraft and Rockets*, Vol. 7, No. 2, Feb. 1970, pp. 119–125.

Prediction of Pressure during Evacuation of Multilayer Insulation

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Nomenclature

c = constant in Eq. (10)
f = specular reflection coefficient
k = constant in Eq. (10)
L = evacuation flow length
M = molecular weight
P = gas pressure
Q = outgassing rate
R_0 = universal gas constant
T = temperature
v_a = mean molecular velocity
v_0 = gas flow velocity at channel wall
v_x, $\bar{v}_x$ = gas flow, mean gas flow velocity
x,y,z = coordinate system defined in Fig. 1
y_0 = half interlayer separation distance
μ = gas viscosity
ρ = gas density
ε = shear stress proportionability constant
τ = gas flow shear stress
ζ = slip coefficient
φ_{pd}, φ_{dp} = parameters defined by Eqs. (3) and (4)

Subscripts

p = purge gas
d = desorbed gas
m = gas mixture

Introduction

MULTILAYER insulation systems require that the interlayer gas pressure be below about 10^{-4} torr for full insulative effectiveness to be achieved. Below this pressure thermal conduction through the residual gas is negligible in comparison with radiation and solid contact conduction. In space applications the interlayer gas pressure must be reduced from one atmosphere to this operational figure by auxiliary pumping equipment, or by evacuation to the atmosphere during ascent. In some applications it may be necessary to pressurize and re-evacuate the multilayers repeatedly in the course of successive missions. This paper describes an improved analytical procedure for predicting the insulation pressure history.

Previous analyses[1,2] of the evacuation of multilayer insulation systems have assumed a parallel plate isothermal flow model with constant interlayer separation, and have treated flow in the laminar continuum and free molecule regimes separately. Outgassing has not been included, and the gas flow has been assumed to be single component. The analysis presented in this paper uses the same simple geometric model but improves the representation of the gas flow by developing simultaneous flow equations for more than one gas component which include the effect of outgassing and

Received November 1, 1971; revision received January 5, 1972. Development of the analytical method was sponsored by the Lockheed Missile and Space Company. The experimental data presented were obtained under contract NAS 8-20758, NASA Marshall Space Flight Center, under the direction of E. Haskell Hyde. The numerical solution to the flow equations was programmed initially for the TYMSHARE System by Shirley L. McDonald and finally for the UNIVAC 1108 by Danielle J. Colvin.

Index categories: Fuel and Propellant Storage, Transfer and Control Systems; Launch Vehicle or Missile Simulation; Thermal Modeling and Experimental Thermal Simulation.

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